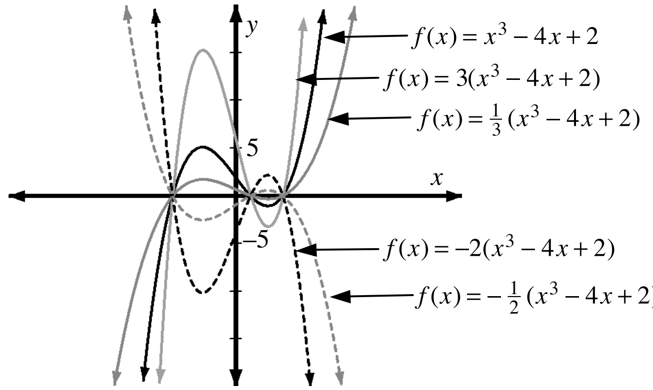
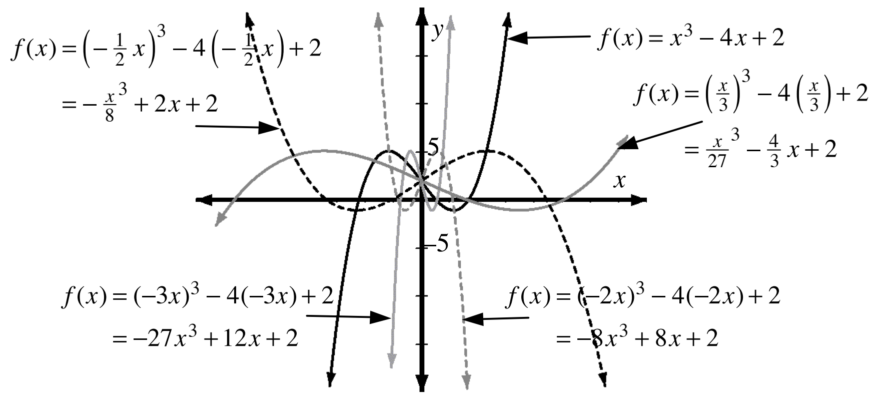
Chapter 3: Exponentials and Logarithms

Lesson 3.1.1

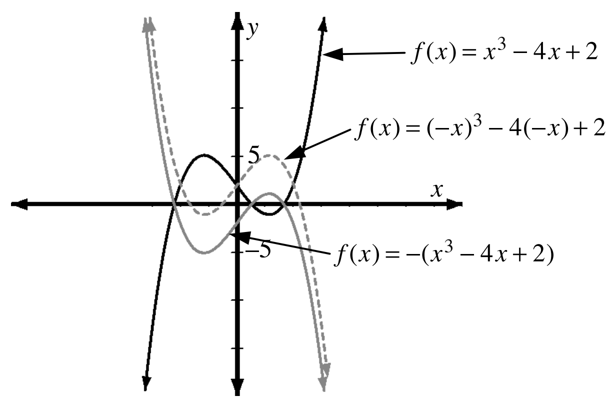
3-1. See graph at right.
$kf(x)$ is a vertical stretch to the graph of $f(x)$ with factor k.



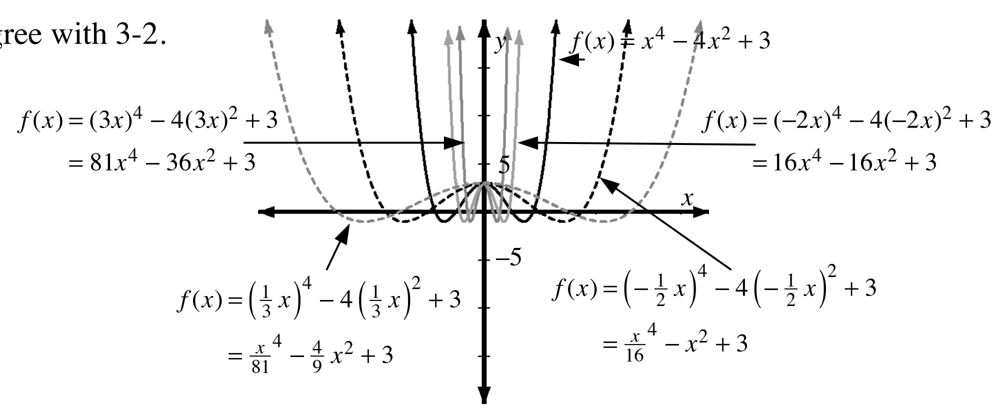
3-2. See graph below. $f(kx)$ is a horizontal stretch to the graph of $f(x)$ with factor $\frac{1}{k}$.



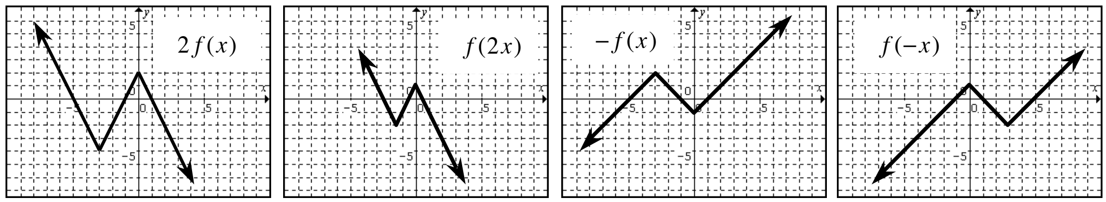
3-3. See graph at right. $-f(x)$ reflects $f(x)$ across the x-axis. $f(-x)$ reflects $f(x)$ across the y-axis.



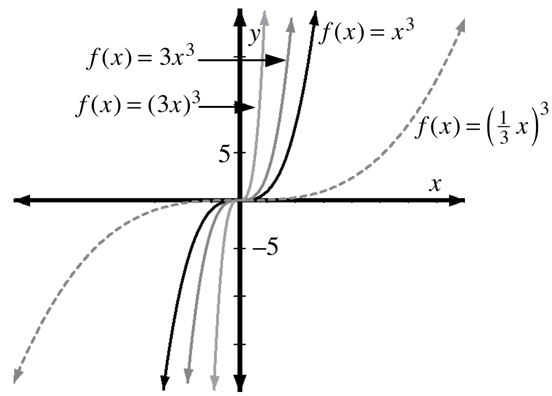
3-4. The results agree with 3-2.



3-5.



Review and Preview 3.1.1



3-6. See graph at right.

a. Vertical stretch

b. Horizontal compression

c. Horizontal stretch

3-7. $g(x)=(3x)^3=3^3x^3=27x^3$, which is a vertical stretch.

3-8. Original $f(x)$:

x	–6	–4	–2	0	2	4	6
$f(x)$	–4	–4	0	4	2	2	2

a. New function:

x	–6	–4	–2	0	2	4	6
$f(x)$	–2	–2	0	2	1.5	1	1

Each y-value is halved, thus this is a vertical compress and the new expression is $\frac{1}{2}f(x)$

b. New function:

x	–6	–4	–2	0	2	4	6
$f(x)$	–4	–4	–4	4	2	2	2

We can see this new function is horizontally compressed with factor $\frac{1}{2}$ thus the new expression is $f(2x)$.

c. New function:

x	–6	–4	–2	0	2	4	6
$f(x)$	2	2	2	4	0	–4	–4

We can see all of the y-values are reversed from the original. Thus, the new expression is $f(-x)$.

3-9.

a. $0.7 = \frac{7}{10}$ so $10^{0.7} = 10^{7/10}$

b. $10^{0.7} = 10^{7/10} = 10^{1/10 \cdot 7} = \left(10^{1/10}\right)^7$ Thus $c = \frac{1}{10}$.

c. $\left(10^{1/10}\right)^7 = \left(\sqrt[10]{10}\right)^7$

d. When not using a calculator, taking the root first makes the number you are raising to a certain power that much smaller, so the work is simplified.

e. $\left(10^{1/10}\right)^7 = \left(\sqrt[10]{10}\right)^7 = 5.012$

f. $10^{0.7} = 5.012$. The answer is the same.

g. $10^{0.71} = 5.129$, too large. $10^{0.69} = 4.898$, too small. $10^{0.699} = 5.000$ is exact up to 0.001.

h. Since we can rewrite $10^x = 5$ as $\log_{10} 5 = x$, calculating $\log 5 = 0.69897...$ gives several more decimal places instantly.

3-10.

$9^x = \left(3^2\right)^x = 3^{2x}$ and $\frac{3}{3^x} = 3 \cdot 3^{-x} = 3^{1-x}$.

Thus $3^{2x} = 3^{1-x}$ and $2x = 1 - x$.

$$3x = 1$$

$$x = \frac{1}{3}$$

Also, $25^x = (5^2)^x = 5^{2x}$ and $\sqrt[5]{5^x} = 5^{x/5}$.

Thus $5^{2x} = 5^{x/5}$ and $2x = \frac{x}{5}$.

The only x that satisfies this equation is $x = 0$.

3-11.

The slope of the line connecting $A(-2,3)$ and $B(4,-5)$ can be found by:

$m = \frac{-5-3}{4+2} = \frac{-8}{6} = -\frac{4}{3}$.

The midpoint of the line can be found by: $M = \left(\frac{-2+4}{2}, \frac{3-5}{2}\right) = (1,-1)$

Point-slope form: $y - y_1 = m(x - x_1)$

The slope of the perpendicular bisector is the negative reciprocal of the line connecting the two points, or $m = \frac{3}{4}$. Using the midpoint $(1,-1)$ in point-slope form we get:

$y + 1 = \frac{3}{4}(x - 1)$

3-12.

a. Clockwise angle is negative. Angle is $-\frac{\pi}{2} - \frac{\pi}{6} = -\frac{2\pi}{3}$ radians away from zero.

b. Counter-clockwise angle is positive. Angle is $2\pi + \frac{\pi}{2} + \frac{2\pi}{6} = \frac{17\pi}{6}$ radians away from zero.

3-13.

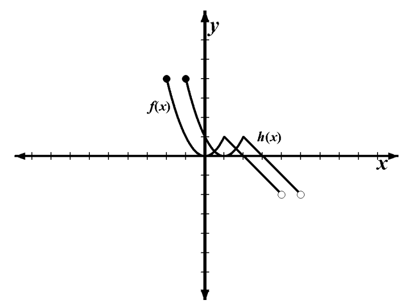
a. ΔPAT is isosceles with $\overline{PA} = \overline{TA}$. Thus, $\left(\overline{PA}\right)^2 + \left(\overline{TA}\right)^2 = \left(\overline{PT}\right)^2 = 8^2 = 64$

$$\begin{aligned} \left(\overline{PA}\right)^2 + \left(\overline{PA}\right)^2 &= \left(\overline{PT}\right)^2 = 64 \\ 2\left(\overline{PA}\right)^2 &= 64 \\ \left(\overline{PA}\right)^2 &= 32 \\ \overline{PA} &= \sqrt{32} = \sqrt{(2\cdot 2)\cdot(2\cdot 2)\cdot 2} \\ &= 4\sqrt{2} \end{aligned}$$

b. $\sin P = \dfrac{\overline{TA}}{\overline{PT}} = \dfrac{\overline{PA}}{\overline{PT}} = \dfrac{4\sqrt{2}}{8} = \dfrac{\sqrt{2}}{2}$

c. $\cos P = \dfrac{\overline{PA}}{\overline{PT}} = \dfrac{4\sqrt{2}}{8} = \dfrac{\sqrt{2}}{2}$

3-14.



a. See graph at right.

b. From the graph, we can see the zeros are $x = 0, 2$ and the range is $-2 < y \le 4$.

c. See graph at right.

d. $h(x) = f(x-1) = \begin{cases} (x-1)^2 & \text{for } -2 \le (x-1) < 1 \\ 2-(x-1) & \text{for } 1 \le (x-1) < 4 \end{cases}$

$= \begin{cases} (x-1)^2 & \text{for } -1 \le x < 2 \\ 3-x & \text{for } 2 \le x < 5 \end{cases}$

e. From part (c) we see the zeros are $x = 1, 3$ and the range is $-2 \le y \le 4$.

Lesson 3.1.2

3-15.

a. $y = a \cdot b^x$ with $(x, y) = (2, 18)$ and $(x, y) = (4, 162)$ we get: $18 = a \cdot b^2$

$162 = a \cdot b^4$

b. In the first equation, solve for a: $a = \frac{18}{b^2}$. Substitute this value for a into the second equation: $162 = \frac{18}{b^2} \cdot b^4 = 18 \cdot b^{-2} \cdot b^4 = 18 \cdot b^2$ or $162 = 18b^2$

c. Solve for b: $a = \frac{18}{b^2}$

$$\begin{aligned} 18 \cdot b^2 &= 162 \\ b^2 &= \tfrac{162}{18} = 9 \\ b &= 3 \end{aligned}$$

d. Use $b = 3$ to solve for a: $a = \frac{18}{b^2} = \frac{18}{3^2} = \frac{18}{9} = 2$

e. The final equation $y = a \cdot b^x$ can be written as: $y = 2 \cdot 3^x$.

3-16.

t	0	1	2	3	4	5	6
$S(t)$	36g	18g	9g	4.5g	2.25g	1.125g	0.563g

3-17.

Using $S(t) = A \cdot b^t$ and the points $(t, S(t)) = (0, 36)$ and $(t, S(t)) = (1, 18)$ (or any two points) we get: $36 = A \cdot b^0 \Rightarrow 36 = A$

$$18 = A \cdot b^1 \Rightarrow 18 = 36 \cdot b \Rightarrow b = \tfrac{1}{2}$$

Thus, $S(t) = A \cdot b^t$ may be written as: $S(t) = 36 \cdot \left(\tfrac{1}{2}\right)^t$.

3-18.

t (min)	0	3	6	9	12	15	18
Number of half-lives	0	1	2	3	4	5	6
$B(t)$	60g	30g	15g	7.5g	3.75g	1.875g	0.938g

3-19.

Using $B(t) = A \cdot b^t$ and the points $(t, B(t)) = (0, 60)$ and $(t, B(t)) = (3, 30)$ we get:

$60 = A \cdot b^0 \Rightarrow A = 60$ $\qquad$ $b^3 = \tfrac{1}{2} \Rightarrow b = \left(\tfrac{1}{2}\right)^{1/3} \Rightarrow b = 0.794$

$30 = A \cdot b^3 \Rightarrow 30 = 60 \cdot b^3$

Thus $B(t) = A \cdot b^t$ may be written as: $B(t) = 60 \cdot (0.794)^t$.

3-20.

a. There will be $\frac{t}{3}$ units in t minutes because $\frac{t}{3} =$ number of half-lives or because each half life is 3 minutes.

b. $B(t) = 60 \cdot (0.5)^{t/3}$ because $C = 60$ is the original amount and because $\frac{t}{3}$ is the number of half-lives in t minutes. During every half-life, the amount of Bromine-85 gets multiplied by $\frac{1}{2} = 0.5$.

3-21.

Using $\text{Total} = C \cdot (\text{percent})^{kt}$ where the original amount saved is $C = \$400$, and the percent of savings left after each year (i.e. $k = 1$) is $1 - 0.1 = 0.9\,\%$ we get: $\text{Total} = 400 \cdot (0.9)^t$. After $t = 5$ years, the total amount left is: $\text{Total} = 400 \cdot (0.9)^5 = \$236.20$.

Review and Preview 3.1.2

3-22.

a. We have two points: $(t,d)=(0,110)$ and $(t,d)=(5,90)$. First, write a system of two equations, then solve for k and m.

$110=km^0 \Rightarrow 110=k$ $\qquad m=\sqrt[5]{\frac{90}{110}}=\sqrt[5]{\frac{9}{11}}\approx 0.961$

$90=km^5 \Rightarrow 90=110m$

The particular equation is $d=110\cdot\left(\sqrt[5]{\frac{9}{11}}\right)^t$

b. $d=110\cdot\left(\sqrt[5]{\frac{9}{11}}\right)^{15}=60.248$ The temperature of the coffee is: $60.248+70=130.248^\circ\text{F}$

3-23.

a. $f(x)=3x^2-x$ reflected over the x-axis is $f(x)=-(3x^2-x)=-3x^2+x$.

b. $f(x)=3x^2-x$ reflected over the y-axis is $f(x)=3(-x)^2-(-x)=3x^2+x$.

3-24.

a. $\sqrt[3]{4}=1.587$ $\qquad$ b. $\left(\sqrt[10]{10}\right)^4=2.512$ $\qquad$ c. $\sqrt[10]{10,000}=2.512$

3-25.

a. Circumference equation: $C=2\pi r=2\pi 10\text{cm}=20\pi\text{cm}$

Area equation: $A=\pi r^2=\pi(10\text{cm})^2=100\pi\text{cm}^2$

b. $\frac{1}{4}$ of the circle is shaded so the area and arc length of the NON-shaded portion are:

$A=\frac{3}{4}(100\pi\text{cm}^2)=75\pi\text{cm}^2$ $\qquad C=\frac{3}{4}(20\pi\text{cm})=15\pi\text{cm}$

3-26.

a. $\frac{2x}{x-y}$ $\qquad$ b. $\frac{5x^2+2x}{x^2-4}$ $\qquad$ c. $\frac{x+2}{x-1}$

3-27.

a. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots+\frac{1}{11}=\sum_{k=3}^{11}\frac{1}{k}$ or $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots+\frac{1}{11}=\frac{1}{1+2}+\frac{1}{2+2}+\frac{1}{3+2}+\cdots+\frac{1}{9+2}=\sum_{k=1}^{9}\frac{1}{k+2}$

b. $\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\cdots+\frac{1}{30}=\frac{1}{2(1)+2}+\frac{1}{2(2)+2}+\frac{1}{2(3)+2}+\cdots+\frac{1}{2(14)+2}=\sum_{k=1}^{14}\frac{1}{2k+2}$

c. $1+8+27+\cdots+216=(1)^3+(2)^3+(3)^3+\cdots+(6)^3=\sum_{k=1}^{6}k^3$

3-28.

a. $A = (1, 4)$ and $B = (5, -2)$. $d = \sqrt{(5-1)^2 + (-2-4)^2} = \sqrt{52}$

b. midpoint $= \left(\frac{1+5}{2}, \frac{4-2}{2}\right) = (3, 1)$

c. The slope of the line is: $m = \frac{-2-4}{5-1} = -\frac{6}{4} = -\frac{3}{2}$

Point-slope form is: $y - y_1 = m(x - x_1)$.

Use point $A = (1, 4)$ to get: $y - 4 = -\frac{3}{2}(x-1)$ or $y = -\frac{3}{2}(x-1) + 4$

Use point $B = (5, -2)$ to get: $y + 2 = -\frac{3}{2}(x-5)$ or $y = -\frac{3}{2}(x-5) - 2$

3-29.

a. At time $t = 0$ Jenny's heart rate was 85 beats per minute (bpm).

b. Jenny's heart rate reached 140 bpm around 17-20 minutes in to her run. Her heart was at least 140 bpm for 5-8 minutes.

c. The area under the curve represents the total number of heartbeats in the 25 minutes she was on the treadmill.

3-30.

a. Let $a = 3, b = \overline{IN}, c = 6$. Using the Pythagorean theorem: $3^2 + b^2 = 6^2$

$b = \sqrt{6^2 - 3^2} = 3\sqrt{3} = \overline{IN}$

b. Using SOH CAH TOA we find $\cos P = \frac{\overline{PI}}{\overline{PN}} = \frac{3}{6} = \frac{1}{2}$

c. $\sin P = \frac{\overline{IN}}{\overline{PN}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

Lesson 3.1.3

3-31.

a. $f(2x) = (2x)^2$ transforms $f(x)$ by a horizontal stretch by $\frac{1}{2}$ or a horizontal compression by 2.

b. $f(2x) = (2x)^2 = 2^2 \cdot x^2 = 4x^2 = ax^2$, thus $a = 4$.

c. $g(x) = ax^2 = 4x^2$ gives a vertical stretch by 4.

d. $f(2x) = (2x)^2 = 2^2 \cdot x^2 = 4x^2$, $g(x) = ax^2 = 4x^2$, $4f(x) = 4(x^2) = 4x^2$

Thus $f(2x) = g(x) = 4f(x)$ because they have the same equation.

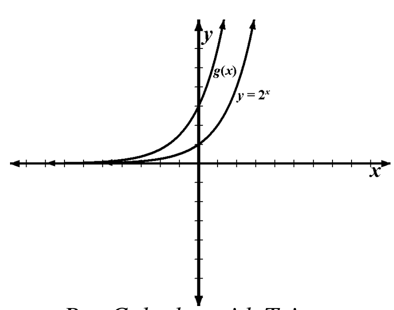
e. Yes, two different transformations may give the same result. This is not true for any function $f(x)$. Example: Exponent Laws: $x^{a+b} = x^a \cdot x^b$

3-32.

a. Letting $y = 2^x$ and $g(x) = 3 \cdot 2^x$ we see that $g(x)$ is a vertical stretch of y by a factor of 3.

b. See graph at right.

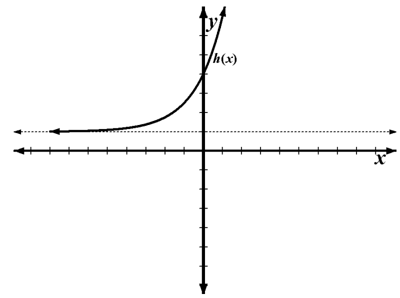
c. The y-intercept of $g(x)$ occurs when $x = 0$ i.e. $g(0) = 3 \cdot 2^0 = 3$. This gives the point $(0, 3)$.



3-33.

a. $h(x)$ is a vertical stretch times 3 and a shift up by 1.

b. See graph at right.



c. From the graph of $h(x)$ we can see that the horizontal asymptote occurs at $y=1$.

d. The y-intercept occurs when $x=0$. i.e. $h(0)=3\cdot 2^0+1=4$. This gives the point $(0,4)$.

e. The y-intercept occurs when $x=0$. i.e. $y=A\cdot 2^x+B$ when $x=0$ gives $y=A\cdot 2^0+B=A+B$. This gives the point $(0,A+B)$.

3-34.

$2\cdot 3^{x+1}=2\cdot 3^x\cdot 3^1=2\cdot 3\cdot 3^x=6\cdot 3^x$

3-35.

a. $y=2^x$ is shifted left 2 units and stretched vertically by 3 to get $y=k(x)$.

b. The y-intercept of $y=k(x)=3\cdot 2^{(x+2)}$ occurs when $x=0$.
$y=k(0)=3\cdot 2^{(0+2)}=3\cdot 2^2=12$ giving the point $(0,12)$.

c. To ensure $m(x)=A\cdot 2^x$ has the same y-intercept, use the point $(x,m(x))=(0,12)$ to get:
$12=A\cdot 2^0=A\Rightarrow A=12$ Thus, $m(x)=12\cdot 2^x$.

d. $k(x)=3\cdot 2^{(x+2)}=3\cdot 2^x\cdot 2^2=3\cdot 2^2\cdot 2^x=12\cdot 2^x=m(x)$ Yes, $k(x)=m(x)$.

3-36.

a. $f(x+2)=3\cdot 4^{(x+2)}=3\cdot 4^2\cdot 4^x=16\cdot(3\cdot 4^x)=16f(x)$

b. $f(x-1)=3\cdot 4^{(x-1)}=3\cdot 4^{-1}\cdot 4^x=\frac{1}{4}\cdot(3\cdot 4^x)=\frac{1}{4}f(x)$

3-37.

$6\cdot 4^{(x+2)}=6\cdot 4^2\cdot 4^x=6\cdot 16\cdot 4^x=96\cdot 4^x=A\cdot 4^x\Rightarrow A=96$

3-39.

a. $y=7^{(x+3)}=7^3\cdot 7^x=343(7^x)$

b. $y=12(5^{x-2})+7=12(5^{-2}\cdot 5^x)+7=\frac{12}{5^2}(5^x)+7=0.48(5^x)+7$

3-40.

a. $5^{2x}=(5^2)^x=25^x$

b. $3^{2x-3}=3^{-3}\cdot 3^{2x}=3^{-3}\cdot(3^2)^x=\frac{1}{27}\cdot(9)^x$

3.1.3 Review and Preview

3-41.

a. $5\cdot 3^{(x+2)}=5\cdot 3^x\cdot 3^2=5\cdot 3^2\cdot 3^x=45\cdot 3^x=A\cdot 3^x \Rightarrow A=45$

b. $\frac{1}{25}\cdot 5^{(x+4)}=\frac{1}{5^2}\cdot 5^x\cdot 5^4=5^{-2}\cdot 5^4\cdot 5^x=5^{-2+4}\cdot 5^x=5^2\cdot 5^x=25\cdot 5^x=A\cdot 5^x \Rightarrow A=25$

c. $16\cdot 2^{(x+4)}=2^4\cdot 2^x\cdot 2^4=2^4\cdot 2^4\cdot 2^x=2^{4+4}\cdot 2^x=2^8\cdot 2^x=256\cdot 2^x=A\cdot 2^x \Rightarrow A=256$

d. $\frac{1}{3}\cdot 3^{(x-2)}=3^{-1}\cdot 3^{-2}\cdot 3^x=3^{-1-2}\cdot 3^x=3^{-3}\cdot 3^x=\frac{1}{3^3}\cdot 3^x=\frac{1}{27}\cdot 3^x=A\cdot 3^x\Rightarrow A=\frac{1}{27}$

3-42.

a. $6 \cdot 2^{(4x+3)} = 6 \cdot 2^{4x} \cdot 2^3 = 6 \cdot 2^3 \cdot 2^{4x} = 48 \cdot (2^4)^x = 48 \cdot (16)^x$

b. $18 \cdot 3^{(3x-4)} = 18 \cdot 3^{3x} \cdot 3^{-4} = 18 \cdot 3^{-4} \cdot 3^{3x} = 18 \cdot \frac{1}{3^4} \cdot (3^3)^x = \frac{18}{81} \cdot (27)^x = \frac{2}{9} \cdot (27)^x$

3-43.

a.
$$\begin{aligned} 8^{(x+3)} &= 32 \\ (2^3)^{(x+3)} &= 2^5 \\ 2^{3x+9} &= 2^5 \\ 3x+9 &= 5 \\ 3x &= -4 \\ x &= -\tfrac{4}{3} \end{aligned}$$

b.
$$\begin{aligned} 27^{2x} &= \left(\tfrac{1}{9}\right)^{(x-1)} \\ (3^3)^{2x} &= \left(\tfrac{1}{3^2}\right)^{(x-1)} \\ 3^{6x} &= (3^{-2})^{(x-1)} \\ 3^{6x} &= 3^{-2x+2} \\ 6x &= -2x+2 \\ 8x &= 2 \\ x &= \tfrac{1}{4} \end{aligned}$$

3-44.

$f(-5) = 2, f(-2) = 2, f(-1) = 2, f(0) = 1, f(1) = 0, f(2) = -1, f(3) = 0, f(4) = 1, f(5) = 2$

$g(-5) = 6, g(-2) = 6, g(-1) = 4, g(0) = 2, g(1) = 0, g(2) = 2, g(3) = 4, g(4) = 6, g(5) = 6$

From inspection we see the minimum is shifted to the left by 1, there is a vertical stretch by 2 and a vertical shift by 2. Thus, $g(x) = 2f(x+1)+2$.

3-45.

a. $\dfrac{t+\frac{1}{t}}{t} = \dfrac{\frac{t^2+1}{t}}{t} = \frac{t^2+1}{t} \cdot \frac{1}{t} = \frac{t^2+1}{t^2}$

b. $\dfrac{1}{x+\frac{y^2}{x}} = \dfrac{1}{\frac{x^2+y^2}{x}} = 1 \cdot \frac{x}{x^2+y^2} = \frac{x}{x^2+y^2}$

c. $\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

d. $\dfrac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

3-46.

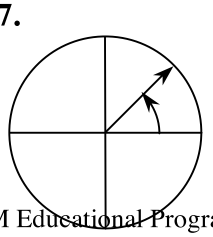
a. $\displaystyle\sum_{i=3}^{6} 4i^3 - 1 = (4 \cdot 3^3 - 1) + (4 \cdot 4^3 - 1) + (4 \cdot 5^3 - 1) + (4 \cdot 6^3 - 1) = 107 + 255 + 499 + 863 = 1724$

b. $0.4\left(\frac{1}{2} + \frac{1}{2.4} + \frac{1}{2.8} + \frac{1}{3.2} + \frac{1}{3.6}\right) = 0.4\left(\frac{1}{1.6+1 \cdot 0.4} + \frac{1}{1.6+2 \cdot 0.4} + \frac{1}{1.6+3 \cdot 0.4} + \frac{1}{1.6+4 \cdot 0.4} + \frac{1}{1.6+5 \cdot 0.4}\right)$

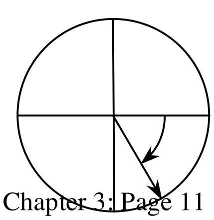
$$= 0.4\sum_{i=1}^{5} \frac{1}{1.6+0.4i}$$

3-47.

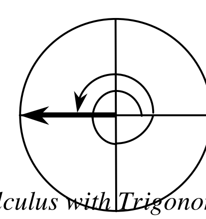
a.



b.



c.



3-48.

a. From Pythagorean theorem we know: $\overline{CA}^2 + \overline{BA}^2 = \overline{CB}^2 \Rightarrow 5^2 + 5^2 = \overline{CB}^2$

$$50 = \overline{CB}^2$$
$$\sqrt{50} = \overline{CB}$$
$$5\sqrt{2} = \overline{CB}$$

b. $\sin C = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

c. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

3-49.

a. $\sum_{n=3}^{7} 4n - 7 = (4 \cdot 3 - 7) + (4 \cdot 4 - 7) + (4 \cdot 5 - 7) + (4 \cdot 6 - 7) + (4 \cdot 7 - 7) = 5 + 9 + 13 + 17 + 21$

b. $\sum_{j=2}^{8} (-1)^j = (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8 = 1 - 1 + 1 - 1 + 1 - 1 + 1$

Lesson 3.2.1

3-50.

$x = 3$, because two of the points have an x-value of 3.

3-51.

a. Yes. $\{(2,6),(-5,1),(-7,4)\}$ is a function because every input (x) has only one output (y).

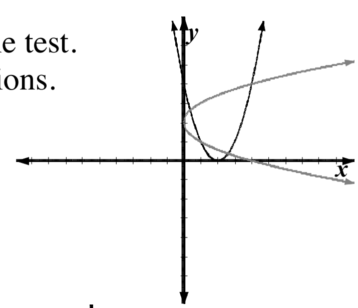
b. Answers vary. Example: $\{(2,6),(-5,1),(-7,4),(2,7)\}$ is not a function because the input 2 corresponds to two outputs: 6 and 7.

3-52.

a. $x = y^2$

b. $x = y^2$ is not a function because it does not pass the vertical line test.

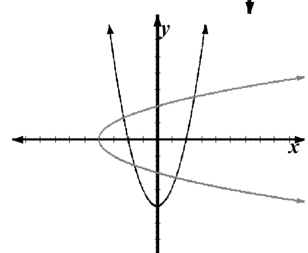
c. $y = |x|$, $y = x^4$, $y = x^3 - 4x$ all have inverses that are not functions.



3-53.

a. See graph above right. If $y = (x-2)^2$, then the inverse relation is $x = (y-2)^2$. Solving for y yields $y = \pm\sqrt{x} + 2$. A range that will make this a function is $y \geq 2$ or $y \leq 2$.

b. See graph below right. If $y = x^2 - 4$, then the inverse relation is $x = y^2 - 4$. Solving for y yields $y = \pm\sqrt{x+4}$. A range that will make this a function is $y \geq 0$ or $y \leq 0$.



3-55.

$g(x)=\frac{2x}{x+2}$. Replace $g(x)$ with y: $y=\frac{2x}{x+2}$. Switch x and y: $x=\frac{2y}{y+2}$.

Solve for y: $x(y+2)=2y$

$$xy+2x=2y$$
$$xy-2y=-2x$$
$$y(x-2)=-2x$$
$$y=\frac{-2x}{x-2}=\frac{2x}{2-x}=g^{-1}(x)$$

3-56.

Yes. f and f^{-1} undo each other. The situation is symmetric.

3-57.

$f^{-1}(x)=x^3+6$. Replace $f^{-1}(x)$ with y: $y=x^3+6$. Switch x and y: $x=y^3+6$.

Solve for y: $x-6=y^3$

$$(x-6)^{1/3}=y$$
$$(x-6)^{1/3}=f(x) \quad \text{or} \quad f(x)=\sqrt[3]{x-6}$$

Review and Preview 3.2.1

3-58.

a. Yes. Switch x and y.

b. $x=|y|$ does not pass the vertical line test.

c. The graph of $y=|x|$ is the reflection of $x=|y|$ over the line $y=x$.

d. $y=|x|=\begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x<0 \end{cases}$

3-59.

$f(x)=\frac{x}{x+1}$

Replace $f(x)$ with y: $y=\frac{x}{x+1}$. Switch x and y: $x=\frac{y}{y+1}$

Solve for y: $(y+1)x=y$

$$yx+x=y$$
$$yx-y=-x$$
$$y(x-1)=-x$$
$$y=\frac{-x}{x-1}\Rightarrow\frac{x}{1-x}=f^{-1}(y)$$

3-60.

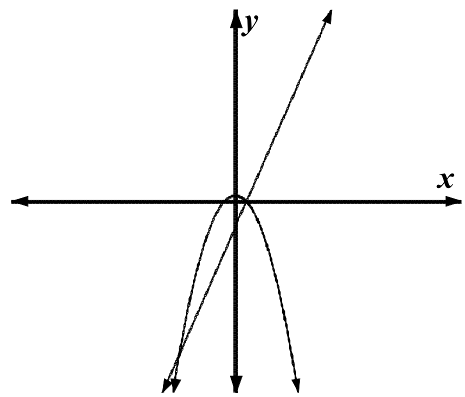
$$\left(\tfrac{1}{125}\right)^{(2x-3)} = \tfrac{1}{25}$$
$$\left(\tfrac{1}{5^3}\right)^{(2x-3)} = \tfrac{1}{5^2}$$
$$5^{-3(2x-3)} = 5^{-2}$$

$$-3(2x-3) = -2$$
$$-6x+9 = -2$$
$$-6x = -11$$
$$x = \tfrac{11}{6}$$

3-61.

a. $x^2 + 2y = 1$ $\quad 2y = 1 - x^2$ $\quad y = \frac{1-x^2}{2}$

$2x - y = 2$ $\quad -y = 2 - 2x$ $\quad y = -2 + 2x$



b. See graph at right. Points of intersections are: $(1, 0)$ and $(-5, -12)$

3-62.

a. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}} = \dfrac{x+y}{\frac{x+y}{xy}} = \frac{x+y}{1} \cdot \frac{xy}{x+y} = xy$

b. $\frac{x}{x+y} - \frac{x-y}{x} = \frac{x^2-(x-y)(x+y)}{x(x+y)} = \frac{x^2-(x^2-y^2)}{x(x+y)} = \frac{y^2}{x(x+y)}$

3-63.

a. $2^x = 8^{(x-1)}$

$2^x = (2^3)^{(x-1)} = 2^{3x-3}$

$x = 3x - 3$

$3 = 2x$

$x = \frac{3}{2}$

b. $4^{-x} = 8^{(x+2)}$

$(2^2)^{-x} = (2^3)^{(x+2)}$

$2^{-2x} = 2^{3x+6}$

$-2x = 3x + 6$

$-6 = 5x$

$x = -\frac{6}{5}$

c. $7^{(1/2x+3)} = 1$

$7^{(1/2x+3)} = 7^0$

$\frac{1}{2}x + 3 = 0$

$\frac{1}{2}x = -3$

$x = -6$

3-64.

Let $\angle A = 35°$. Let B be the point of the first measurement with angle $42°$. Let D be the point at the peak of the mountain. Draw a vertical line from B to create a smaller, right triangle. Let E be the intersection of the vertical line emerging from B.

Find the height of the line $\overline{BE}$ by: $\tan(35) = \frac{BE}{1200}$, $BE = 1200\tan(35) = 840.249$

We also know the following angles:

$\angle ABE = 90°$, $\angle BAE = 35°$ $\qquad \angle BEA = 180° - 90° - 35° = 55°$

$\angle BED = 180° - 55° = 125°$ $\qquad \angle EBD = 180° - 90° - 42° = 48°$

$\angle EDB = 180° - 125° - 48° = 7°$

Solution continues on next page. →

3-64. *Solution continued from previous page.*

Knowing this, we can find BD: $\frac{BD}{\sin(125)} = \frac{840.249}{\sin(7)}$

$$BD = \frac{840.249}{\sin(7)} \cdot \sin(125) = 5,647.784$$

Now we can find h: $\sin(42) = \frac{h}{5647.784}$

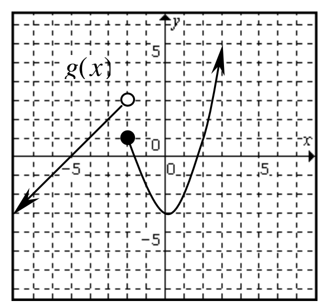
$$h = 5647.784 \sin(42)$$
$$h = 3,779.11 \text{ ft}$$

3-65.

a. For the left piece of $f(x)$ the slope of the line is $m = 1$ and the y-intercept is $b = 2$. Using the equation for a line ($y = mx + b$) we get: $f(x) = x + 2$ for $x < 0$.
The right piece of $f(x)$ is a parabola shifted to the right by 2 and down by 4. Using the equation for a parabola $y = (x+h)^2 + k$ and letting $h = -2$ and $k = -4$ to get the appropriate shifts, we get: $f(x) = (x-2)^2 - 4$ for $x \geq 0$.

Combine these to get: $f(x) = \begin{cases} x+2 & \text{for } x < 0 \\ (x-2)^2 - 4 & \text{for } x \geq 0 \end{cases}$

b. See graph at right.



$$g(x) = f(x+2) + 1$$
$$= \begin{cases} (x+2)+2+1 & \text{for } x+2 < 0 \\ ((x+2)-2)^2 - 4 + 1 & \text{for } x+2 \geq 0 \end{cases}$$
$$= \begin{cases} x+5 & \text{for } x < -2 \\ x^2 - 3 & \text{for } x \geq -2 \end{cases}$$

3-66.

To find b so that there is only one intersection point, the discriminant must equal 0. The discriminant is the part of the quadratic formula that is inside of the Square root.

$$2x + b = x^2 - 6x + 6$$
$$x^2 - 8x + (6 - b) = 0$$

Using the Quadratic Formula: $x_{1,2} = \frac{8 \pm \sqrt{8^2 - 4(6-b)}}{2}$

$$= \frac{8 \pm \sqrt{40 + 4b}}{2}$$

There is only one intersection point if: $\sqrt{40 + 4b} = 0$

$$40 = -4b$$
$$b = -10$$

Lesson 3.2.2

3-67.

a.

x	y
625	4
1/5	–1
5	1
125	3
1	0
-1	Undefined
0	Undefined
25	2
1/25	–2
2.236	1/2

b. $x = 5^y$

3-68.

a. $\log_5(625) = 4$ because $5^4 = 625$.

b. $y = \log_5(x)$

3-69.

a. $\log_5(625) = 4$

b. $\log_5(125) = 3$ because $5^3 = 125$.

c. $\log_5(25) = 2$ because $5^2 = 25$.

3-70.

a. $\log_2 8 = 3$ because $2^3 = 8$.

b. $\log_6(1296) = 4$ because $6^4 = 1296$.

c. $\log_7\left(\frac{1}{49}\right) = -2$ because $7^{-2} = \frac{1}{49}$.

d. $\log_9 3 = \frac{1}{2}$ because $9^{1/2} = 3$.

3-71.

a. $y = 7^x$ can be rewritten as $x = \log_7 y$.

b. $\log_4 x = y$ can be rewritten as $4^y = x$.

c. $11^y = x$ can be rewritten as $y = \log_{11} x$.

d. $W^k = B$ can be rewritten as $\log_W B = K$.

e. $K = \log_W B$ can be rewritten as $B = W^K$.

f. $\log_{1/3} P = Q$ can be rewritten as $P = \left(\frac{1}{3}\right)^Q$.

3-72.

$\log 100 = 2, \log(3.162) = 0.5,$ etc indicates that the calculator uses base 10 because: $10^2 = 100, 10^{0.5} = 3.162$, etc.

Review and Preview 3.2.2

3-73.

$b > 0$ because $\log_b x = \frac{\log x}{\log b}$ and $\log(b)$ can only take values $b > 0$.

3-74.

a. From Pythagorean theorem: $(RA)^2 + n^2 = (2n)^2$

$$(RA)^2 = (2n)^2 - n^2$$
$$(RA)^2 = 4n^2 - n^2$$
$$(RA)^2 = 3n^2$$
$$RA = n\sqrt{3}$$

b. *i.* $\sin R = \frac{n}{2n} = \frac{1}{2}$

ii. $\cos R = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$

iii. $\tan R = \frac{n}{n\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c. $\frac{\sin R}{\cos R} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan R$

3-75.

a. $\log_5 5 = 1$ because $5^1 = 5$

b. $\log_5 1 = 0$ because $5^0 = 1$

c. $\log_5 \left(\frac{1}{5}\right) = -1$ because $5^{-1} = \frac{1}{5}$

3-76.

a. $\log_3 81 = 4$ because $3^4 = 81$

b. $3^2 = 9$. 9 goes into the $\log_3$ machine. $\log_3 9 = 2$ because $3^2 = 9$.

c. $3^x = y$. $\log_3 y = \log_3 3^x = x$ because $3^x = 3^x$.

d. $\log_3 x = y$

e. $x = 3^y$

f. Yes. $3 > 0$, so $y = \log_3 x$ means $x = 3^y$.

3-77.

Area of a triangle is $A = \frac{1}{2} b \cdot h$ where $b = 15\text{cm}$. Let $\angle A = 36°, \angle B = 25°$ then, if we call this triangle ΔABC then $\angle C = 180° - 36° - 25° = 119°$.

Using the sine rule we know: $\frac{\overline{CB}}{\sin(36)} = \frac{15}{\sin(119)}$ $\overline{CB} = \frac{15}{\sin(119)} \sin(36) = 10.081\text{cm}$

From C, draw a line perpendicular to $\overline{AB}$. Let D be the point where this line intersects $\overline{AB}$. Because ΔBCD is a right triangle we know: $\sin(25) = \frac{\overline{CD}}{10.081}$

$\overline{CD} = \sin(25) \cdot 10.081 = 4.260\text{cm}$.

Thus, the height of the triangle is $h = 4.260cm$ and the area is:

$A = \frac{1}{2}(15\text{cm})(4.260\text{cm}) = 31.952\text{cm}^2$

3-78.

a. To break the interval $(2,5)$ into six pieces, the width, x, of the rectangles will be $(5-2)\frac{1}{x}=6;\ x=\frac{1}{2}$. The area of each rectangle is $A=b\cdot h$ where $b=\frac{1}{2}$ and $h=3x^2$.

The area in sigma notation using left-endpoint rectangles is: $\frac{1}{2}\sum\limits_{k=0}^{5}3(0.5k+2)^2$

b. To use right-endpoint rectangles: $\frac{1}{2}\sum\limits_{k=1}^{6}3(0.5k+2)^2$

c. To use midpoint rectangles: $\frac{1}{2}\sum\limits_{k=0}^{5}3(0.5k+2.25)^2$

d. To estimate the area using trapezoids, average the left- and right-endpoint results.

3-79.

$f(-4)=-6, f(-3)=-1, f(-2)=2, f(-1)=3, f(0)=2, f(1)=-1, f(2)=1, f(3)=3, f(4)=5$
$g(-1)=7, g(0)=2, g(1)=-1, g(2)=-2, g(3)=-1, g(4)=2, g(5)=0, g(6)=-2, g(7)=-4$
It is clear from the graph that $g(x)$ is a vertically flipped version of $f(x)$. The function is then shifted to the right by 3 and up by 1 to get: $g(x)=-f(x-3)+1$.

3-80.

a.
$$\begin{aligned}8(4)^x &= \sqrt[3]{1/2^x}\\ (2^3)(2^2)^x &= (2^{-x})^{1/3}\\ 2^{3+2x} &= 2^{-x/3}\\ 3+2x &= -\tfrac{x}{3}\\ 9+6x &= -x\\ 7x &= -9\\ x &= -\tfrac{9}{7}\end{aligned}$$

b.
$$\begin{aligned}\left(\tfrac{1}{25}\right)^{(x+1)} &= \sqrt{125^x}\\ (5^{-2})^{(x+1)} &= ((5^3)^x)^{1/2}\\ 5^{-2x-2} &= 5^{3x/2}\\ -2x-2 &= \tfrac{3x}{2}\\ -4x-4 &= 3x\\ -4 &= 7x\\ x &= -\tfrac{4}{7}\end{aligned}$$

3-81.

a. $f(x)=g(x)$ when $x^2=-x^2+2x+4 \Rightarrow 2x^2-2x-4=0$.

$$x=\frac{2\pm\sqrt{(-4)^2-4(2)(-4)}}{2(2)}=\frac{2\pm\sqrt{36}}{4}=\frac{2\pm 6}{4}=-1,2$$

When $x=-1$, $f(-1)=g(-1)=1$. When $x=2$, $f(2)=g(2)=4$. Thus, the two intersection points are $(-1,1)$ and $(2,4)$.

b. $\int\limits_{-1}^{2}(-x^2+2x+4-x^2)dx=9$ units

Lesson 3.2.3

3-82.

a. Let $y = \log_2 x$. This can be rewritten as $2^y = x$. The inverse of this function is: $2^x = y$.

b. The domain of $2^x = y$ is all real numbers, the range is $y > 0$.

c. See graph at right.

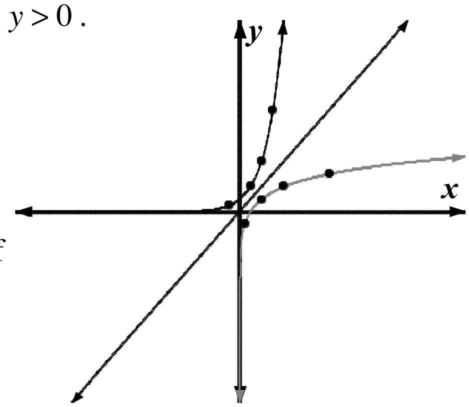
d. See graph at right.

e. See graph at right.

f. Domain: $x > 0$, Range: all real numbers.

g. Domain: $x > 0$, Range: all real numbers.

h. The graphs have the same general shape. The graph of $y = \log_2 x$ is vertically stretched. They have the point $(1,0)$ in common.



3-83.

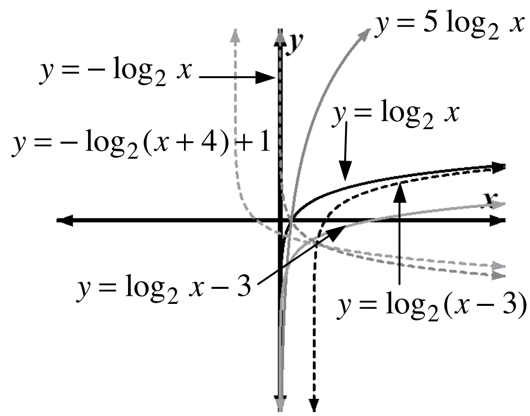
a. To get $y = 5\log_2 x$, stretch $y = \log_2 x$ vertically by a factor of 5. The vertical asymptote is $x = 0$.

b. To get $y = \log_2 x - 3$, shift $y = \log_2 x$ down 3 units. The vertical asymptote is $x = 0$.

c. To get $y = \log_2(x-3)$, shift $y = \log_2 x$ 3 units to the right. The vertical asymptote is $x = 3$.

d. To get $y = -\log_2 x$, reflect $y = \log_2 x$ across the x-axis. The vertical asymptote is $x = 0$.

e. To get $y = -\log_2(x+4)+1$, shift $y = \log_2 x$ 4 units to the left then reflect $y = \log_2 x$ across the x-axis, and shift if up by 1 unit. The vertical asymptote is $x = -4$.



3-84.

a. Domain: $x > 0$. Range: all real numbers.

b. Domain: $x > 0$. Range: all real numbers.

c. Domain: $x > 3$. Range: all real numbers.

d. Domain: $x > 0$. Range: all real numbers.

e. Domain: $x > -4$. Range: all real numbers.

3-85.

$f(x) = \log_2 x = 10$ when $x = 2^{10} = 1024$.

3-86.

$f(x) = \log_2 x = 20$ when $x = 2^{20} = 1{,}048{,}576$.

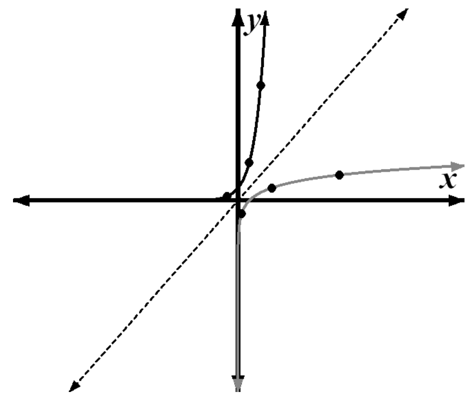
3-87.

To get a y-value of 20 you would have to go $1,048,576$ units or

$0.25\,\frac{\text{inches}}{\text{unit}} \cdot 1,048,576 \text{ units} = 262,144 \text{ inches}$ or $= \frac{1 \text{ foot}}{12 \text{ inches}} \cdot 262,144 \text{ inches} = 21,845.333 \text{ feet}$

or $= \frac{1 \text{ mile}}{5280 \text{ feet}} = 21,845.333 \text{ feet} = 4.137 \text{ miles}$

Review and Preview 3.2.3



3-88.

See graph at right.

$f(x)$: Domain: all real numbers. Range: $y > 0$.

There are no zeros.

$g(x)$: Domain: $x > 0$. Range: all real numbers.

Zeroes: $x = 1$.

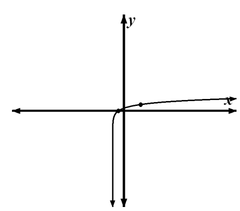
3-89.

$f(x) = 3^x$ and $f^{-1}(x) = \log_3 x$

a. $f(4) = 3^4 = 81$ b. $f^{-1}(81) = \log_3 81 = 4$ c. $f(-2) = 3^{-2} = \frac{1}{9}$

d. $f^{-1}\left(\frac{1}{9}\right) = \log_3 \frac{1}{9} = -2$ e. $f\left(\frac{1}{2}\right) = 3^{1/2} = \sqrt{3}$ f. $f^{-1}\left(\sqrt{3}\right) = \log_3 \sqrt{3} = \frac{1}{2}$

3-90.



See graph at right.

$f(x) = \log_5(2 + x)$

Domain: $x > -2$ Range: $-\infty < y < \infty$

3-91.

a. $4x^2 - y^2 = (2x + y)(2x - y)$ b. $9z^2 - y^2 = (3z + y)(3z - y)$

3-92.

$f(x) = \frac{x+3}{2x}$. Let $y = \frac{x+3}{2x}$. Switch x and y: $x = \frac{y+3}{2y}$.

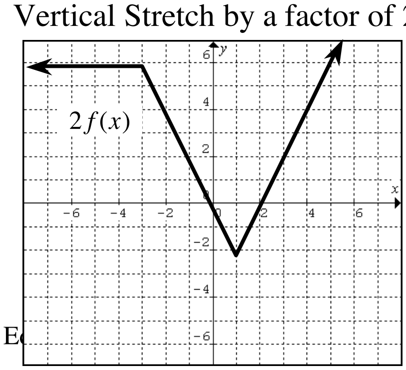
Solve for y: $2xy = y + 3$ $\quad y = \frac{3}{2x-1}$

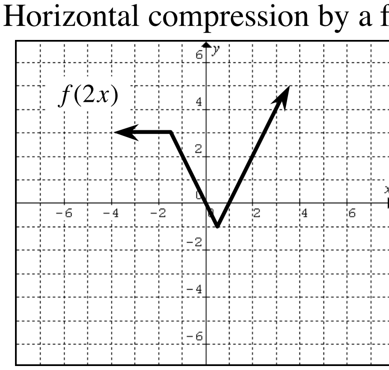
$2xy - y = 3 \quad f^{-1}(x) = \frac{3}{2x-1}$

$y(2x - 1) = 3$

3-93.

a. Vertical Stretch by a factor of 2. b. Horizontal compression by a factor of 2.

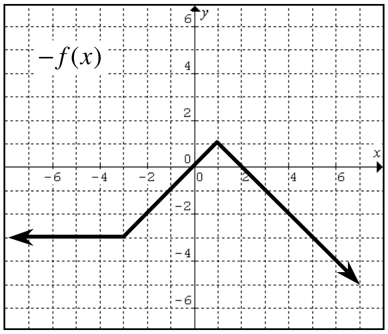




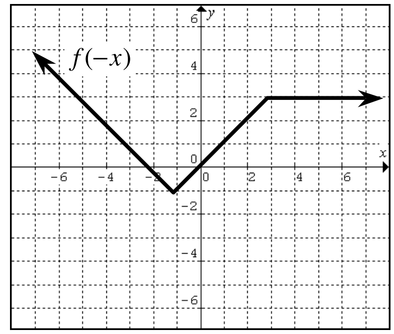
Solution continues on next page. →

3-93. *Solution continued from previous page.*

c. Flipped over the *x*-axis.



d. Flipped over the *y*-axis.



3-94.

a. $-\left(\pi+\frac{\pi}{6}\right)=-\frac{7\pi}{6}$

b. $\pi+\frac{\pi}{2}+\frac{\pi}{4}=\frac{7\pi}{4}$

3-95.

a. As x increases from 2 to 4, y increases from 36 to 81. Thus, the function is increasing.

b. Use the form $y=A\cdot b^x$ and the points $(2,36)$ and $(4,81)$. First, substitute in $(2,36)$: $36=A\cdot b^2$. Solve for A: $36\cdot b^{-2}=A$. Use this value of A when substituting in the point $(4,81)$: $81=36\cdot b^{-2}\cdot b^4$.

Solve for *b*:

$$\begin{aligned} 81&=36\cdot b^2\\ 2.25&=b^2\\ \sqrt{2.25}&=b\\ 1.5&=b \end{aligned}$$

Use this value to solve for A:

$$\begin{aligned} 36\cdot b^{-2}&=A\\ 36\cdot(1.5)^{-2}&=A\\ 16&=A \end{aligned}$$

Thus, the exponential function that passes through the two points is: $y=f(x)=16(1.5)^x$.

c. To have a horizontal asymptote of $y=20$, the function in part (b) must be shifted up by 20: $f(x)=16(1.5)^x+20$

3-96.

a. $\left(1-\frac{x}{y}\right)\left(1+\frac{x}{y}\right)$

$1+\frac{x}{y}-\frac{x}{y}-\frac{x^2}{y^2}$

$1-\frac{x^2}{y^2}$

$\frac{y^2-x^2}{y^2}$

b. $\frac{\frac{y}{x}-\frac{x}{y}}{\frac{y}{x}+\frac{x}{y}}+1$

$\frac{\frac{y^2-x^2}{xy}}{\frac{y^2+x^2}{xy}}+1$

$\frac{y^2-x^2}{x^2+y^2}+1$

$\frac{\left(y^2-x^2\right)+\left(x^2+y^2\right)}{x^2+y^2}$

$\frac{2y^2}{x^2+y^2}$

Lesson 3.3.1

3-97.

a. Yes the entries are correct.

b. No, this relation does not hold.

c. $\log 2 + \log 3 = 0.301 + 0.477 = 0.778$
$\log 6 = 0.778$
$\log 2 + \log 3 = \log 6$

d. $\log 2 + \log 4 = 0.301 + 0.602 = 0.903$
$\log 8 = 0.903$
$\log 2 + \log 4 = \log 8$

e. $\log 3 + \log 4 = 0.477 + 0.602 = 1.079$
$\log 12 = 1.079$
$\log 3 + \log 4 = \log 12$

f. $\log x + \log y = \log xy$

3-98.

a. $\log_3 9 + \log_3 27 = 5$ and $\log_3(9 \cdot 27) = \log_3 243 = 5$, thus, $\log_3 9 + \log_3 27 = \log_3(9 \cdot 27)$.

b. $\log_4 8 + \log_4 4 = \frac{5}{2}$ and $\log_4(8 \cdot 4) = \log_3 32 = \frac{5}{2}$, thus, $\log_4 8 + \log_4 4 = \log_4(8 \cdot 4)$.

c. Both sides $= -3$

3-99.

Problem 3-98 does not prove that $\log_b x + \log_b y = \log_b xy$. Showing a relation holds for three cases does not show it holds for all cases. This pattern may break down with more examples. We have not shown why this pattern is always true, yet.

3-100.

a. $\log 6 - \log 2 = 0.778 - 0.301 = 0.477$ and $\log 3 = 0.477$, thus, $\log 6 - \log 2 = \log 3$.

b. $\log 8 - \log 4 = 0.903 - 0.602 = 0.301$ and $\log 2 = 0.301$, thus, $\log 8 - \log 4 = \log 2$.

c. $\log x - \log y = \log \frac{x}{y}$

3-101.

Let $M = 8$ and $N = 2$. Then $\log\left(\frac{8}{2}\right) = \log 4 = 0.602$

$$\frac{\log 8}{\log 2} = \frac{0.903}{0.301} = 3$$

$$\log\left(\frac{8}{2}\right) \neq \frac{\log 8}{\log 2}$$

3-102.

a. $2\log 2 = 2(0.301) = 0.602 = \log 4$

b. $3\log 2 = 3(0.301) = 0.903 = \log 8$

c. $4\log 2 = 4(0.301) = 1.204 = \log 16$

d. $n\log x = \log x^n$

e. $-2\log 5 = \log 5^{-2} = \log \frac{1}{25}$

f. $0.5\log 64 = \log 64^{0.5} = \log 8$

3-104.

Let $\log x = N$ so that $10^N = x$.
Let $\log y = M$ so that $10^M = y$.

$$\left(\frac{x}{y}\right) = \left(\frac{10^N}{10^M}\right) = 10^{N-M}$$

$$\log\left(\frac{x}{y}\right) = \log\left(10^{N-M}\right) = N - M$$

$$\log\left(\frac{x}{y}\right) = \log x - \log y$$

3-105.

Let $\log x = M$ so that $10^M = x$. $\quad x^n = \left(10^M\right)^n = 10^{Mn}$

$$\log x^n = \log\left(10^{Mn}\right) = Mn$$

$$\log x^n = n \log x$$

Review and Preview 3.3.1

3-106.

a. $\ln(3 \cdot 2) = 1.792$ and $\ln 3 + \ln 2 = 1.792$. Thus, $\ln(3 \cdot 2) = \ln 3 + \ln 2$.

$\ln \frac{3}{2} = 0.405$ and $\ln 3 - \ln 2 = 0.405$. Thus, $\ln \frac{3}{2} = \ln 3 - \ln 2$.

$\ln 3^2 = 2.197$ and $2 \ln 3 = 2.197$. Thus, $\ln 3^2 = 2 \ln 3$.

The three log laws work in these and all other cases.

b. $\log_b b = 1$ because $b^1 = b$.

c. Example, $\ln 3 = 1.099$. $x^{1.099} = 3$ when $x = 2.718$.

The base of the natural logarithm is $x = 2.718$.

3-107.

a. $\log_3 81 = 4$ because $3^4 = 81$.

b. $\log_5 \sqrt{5} = \frac{1}{2}$ because $5^{1/2} = \sqrt{5}$.

c. $\log_4 \frac{1}{16} = -2$ because $4^{-2} = \frac{1}{16}$.

3-108.

a. $\log 4 + \log 2 - \log 5 = \log(4 \cdot 2) - \log 5 = \log 8 - \log 5 = \log \frac{8}{5}$

b. $\log_2 M + 2\log_2 N = \log_2 M + \log_2 N^2 = \log_2\left(M \cdot N^2\right) = \log_2 MN^2$

3-109.

$\log_4 x = y$ can be rewritten as $4^y = x$. $4^y = 2^{2y} = x$ can be rewritten as $\log_2 x = 2y$.

Solving for y gives: $\frac{1}{2}\log_2 x = y$.

3-110.

a. $2\pi + \frac{\pi}{2} + \frac{\pi}{6} = \frac{8\pi}{3}$

b. $-\frac{\pi}{2} - \frac{2\pi}{6} = -\frac{5\pi}{6}$

3-111.

a. $f(x) = y = \sqrt[3]{3x-5}$. Switch x and y: $x = \sqrt[3]{3y-5}$. Solve for y: $\quad x^3 = 3y - 5$

$$x^3 + 5 = 3y$$

$$\frac{x^3+5}{3} = y = f^{-1}(x)$$

Solution continues on next page. →

3-111. *Solution continued from previous page.*

b. $g(x) = y = \frac{2x-1}{3-x}$. Switch x and y: $x = \frac{2y-1}{3-y}$. Solve for y:

$$\begin{aligned} x(3-y) &= 2y-1 \\ 3x - xy + 1 &= 2y \\ 3x+1 &= 2y + xy \\ 3x+1 &= y(2+x) \\ y &= \tfrac{3x+1}{2+x} = g^{-1}(x) \end{aligned}$$

c. $h(x) = y = \log_3(x-1)$. Switch x and y: $x = \log_3(y-1)$.

Solve for y: $3^x = y-1$

$$y = 3^x + 1 = h^{-1}(x)$$

3-112.

a. Since this is a right triangle we know: $AC^2 + BC^2 = AB^2$ or

$$\begin{aligned} (n\sqrt{2})^2 + (n\sqrt{2})^2 &= AB^2 \\ 2n^2 + 2n^2 &= AB^2 \\ 4n^2 &= AB^2 \\ 2n &= AB \end{aligned}$$

b. Using the law of sines: $\frac{n\sqrt{2}}{\sin A} = \frac{2n}{\sin 90^\circ}$

$$\begin{aligned} 2n \sin A &= n\sqrt{2} \\ \sin A &= \tfrac{n\sqrt{2}}{2n} = \tfrac{\sqrt{2}}{2} \end{aligned}$$

c. $\sin A = \frac{\sqrt{2}}{2}$ no matter the value of n.

d. $\sin 45^\circ = \frac{\sqrt{2}}{2}$ always.

e. $\cos 45^\circ = \cos A = \frac{n\sqrt{2}}{2n} = \frac{\sqrt{2}}{2}$ always.

3-113.

$f(x)$ will be continuous at $x = 0$ if $ax^2 + b = 2ax + 5$ when $x = 0$ or $b = 5$. $f(x)$ will be continuous at $x = 1$ if $2ax + 5 = 3x - b$ when $x = 1$ or $2a + 5 = 3 - b = 3 - 5 = -2$

$$\begin{aligned} 2a &= -2 - 5 \\ a &= -\tfrac{7}{2} \end{aligned}$$

3-114.

a. $y = 16\left(\frac{1}{2}\right)^{2x+1} = 16\left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{1} = 8(\left(\frac{1}{2}\right)^{2})^{x} = 8\left(\frac{1}{4}\right)^{x}$

b. $y = 100(25)^{1/2\,x-1} = 100(25)^{1/2\,x}(25)^{-1} = 4(25^{1/2})^{x} = 4(5)^{x}$

Lesson 3.3.2

3-115.

Given equation $1.05^x = 2$

Take logs of both sides $\log 1.05^x = \log 2$

Power Law for logs $x \log 1.05 = \log 2$

Divide both sides by $\log 1.05$ $x = \frac{\log 2}{\log 1.05}$

Use calculator to evaluate $x = 14.207$

3-116.

Let $x =$ # of years and $a =$ prices. Then prices have doubled at $2a$. The inflation function is $a(1+0.05)^x$. Prices have doubled when $2a = a(1.05)^x \Rightarrow 2 = (1.05)^x$.

$\log 2 = \log(1.05)^x$ $x = 14.207$ years.

$\log 2 = x \log(1.05)$

$x = \frac{\log 2}{\log(1.05)} = 14.207$

3-117.

$40^1 = 40$ and $40^2 = 1600$, so maybe $40^{1.5} = 400$? Actual answer: $x = 1.624$

$\log_2 8 = 3$ because $2^3 = 8$. $\log_3 27 = 3$ because $3^3 = 27$.

3-118.

a. Yes he is right. If $\log_6 260 = x$ then we know that $6^x = 260$.

b. We currently do not know how to find $\log_6 260$.

c. $6^x = 260$

$\log 6^x = \log 260$

$x \log 6 = \log 260$

$x = \frac{\log 260}{\log 6}$

$x = 3.103$

d. Yes, because the x-value that satisfies $6^x = 260$ also satisfies $x = \log_6 260$.

3-119.

a. $x = \log_3 11$ can be rewritten as $3^x = 11$. Using the previous method:

$3^x = 11$

$\log 3^x = \log 11$

$x \log 3 = \log 11$

$x = \frac{\log 11}{\log 3}$

$x = 2.183$

b. To graph $y = \log_3 x$, we can use $y = \frac{\log x}{\log 3}$.

c. The range of $y = \log_3 x$ is all real numbers. The calculator plots this function by plotting discrete points and then connecting them with segments. The next x-value to the left of $x = 0.21$ is $x = 0$ and $\log_3 0$ is undefined, so the graph stops at $x = 0.21$.

3-120.

a. $\log_3 8 \approx 1.9w$ because $3^2 = 9$ so $3^{1.9} \approx 8$.

b. Let $\log_3 8 = x$. This can be rewritten as $3^x = 8$. Taking the log of both sides as before:

$$\log 3^x = \log 8$$
$$x \log 3 = \log 8$$
$$x = \frac{\log 8}{\log 3}$$

Now set the two values of x equal to each other: $x = \log_3 8 = \frac{\log 8}{\log 3} w$

c. $\frac{\log 8}{\log 3} = 1.893$

d. $\log_3 8 = 1.893$ can be rewritten as: $3^{1.893} = 8$. It checks.

3-121.

a. The initial amount is 20mg, hence $20(0.9)^0 = 20$. Also, each hour, 90% of what was there the hour before remains.

b. $20(0.9)^t = 12$ when $(0.9)^t = \frac{12}{20} = 0.6$. Taking the log of both sides:

$$\log(0.9)^t = \log(0.6)$$
$$t \log(0.9) = \log(0.6)$$
$$t = \frac{\log(0.6)}{\log(0.9)}$$

c. $t = \frac{\log(0.6)}{\log(0.9)} = 4.848$ or $t = 4$ hours 51 mintues.

Review and Preview 3.3.3

3-122.

a. Estimate $20^x = 316$ has a solution $x \approx 1.9$ because $20^2 = 400$.

Actual solution:

$$20^x = 316$$
$$x \log 20 = \log 316$$
$$x = \frac{\log 316}{\log 20} = 1.921$$

b. Estimate $(7.3)^x = 4.81$ has a solution $x \approx 0.6$ because $(7.3)^1 = 7.3$.

Actual solution:

$$(7.3)^x = 4.81$$
$$x \log(7.3) = \log(4.81)$$
$$x = \frac{\log(4.81)}{\log(7.3)} = 0.790$$

c. Estimate $160(0.5)^x = 8$ has a solution $x \approx 4.1$ because when $x = 4$: $160(0.5)^4 = 160\left(\frac{1}{2}\right)^4 = \frac{160}{2^4} = \frac{160}{16} = 10$

Actual Solution:

$$160(0.5)^x = 8$$
$$(0.5)^x = \frac{8}{160} = 0.05$$
$$x \log(0.5) = \log(0.05)$$
$$x = \frac{\log(0.05)}{\log(0.5)} = 4.322$$

3-123.

a. $\log_2 x^3 = 6$

$2^6 = x^3$

$x = (2^6)^{1/3} = 2^2 = 4$

b. $\log_4 x + \log_4 3 = 2$

$\log_4 3x = 2$

$4^2 = 3x$

$x = \frac{4^2}{3} = \frac{16}{3}$

3-124.

a. $200(1.05)^x = 1000$

$(1.05)^x = \frac{1000}{200} = 5$

$x\log(1.05) = \log 5$

$x = \frac{\log 5}{\log 1.05} = 32.987$

b. $20(2.5)^x - 400 = 600$

$20(2.5)^x = 1000$

$2.5^x = 50$

$x\log 2.5 = \log 50$

$x = \frac{\log 50}{\log 2.5} = 4.269$

3-125.

a. $2\log m - 3\log n + \frac{1}{2}\log p$

$= \log m^2 - \log n^3 + \log p^{1/2}$

$= \log \frac{m^2\sqrt{p}}{n^3}$

b. $\frac{1}{2}(\log a + 2\log b - 3\log c)$

$= \frac{1}{2}(\log a + \log b^2 - \log c^3)$

$= \frac{1}{2}\left(\log \frac{ab^2}{c^3}\right) = \left(\log \frac{ab^2}{c^3}\right)^{1/2}$

$= \log \sqrt{ab^2/c^3}$

3-126.

$x^{1.05} = 2$

$(x^{1.05})^{1/1.05} = 2^{1/1.05}$

$x = 1.935$

3-127.

b. $\log_b 2 = 0.693147$ can be rewritten as $b^{0.693147} = 2$.

Solving for b: $\left(b^{0.693147}\right)^{1/0.693147} = 2^{1/0.693147}$, $b = 2.71828$

3-128.

a. Using Pythagorean theorem we get: $PQ^2 + n^2 = (2n)^2 = (2n)^2 - n^2 = 4n^2 - n^2 = 3n^2$

$\overline{PQ} = \sqrt{3n^2} = n\sqrt{3}$

b. $\sin P = \frac{n}{2n} = \frac{1}{2}$

c. $\cos P = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$

d. From the Law of Sines we know: $\frac{2n}{\sin Q} = \frac{n}{\sin P}$

We also know $\sin Q = \sin(90) = 1$ so: $2n = \frac{n}{\sin P} \Rightarrow \sin P = \frac{1}{2} \Rightarrow P = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

e. The ratios do not depend on the value of n, so they will always be the same as those found in (b) and (c).

3-129.

a. $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

b. $-2\pi - \pi - \frac{\pi}{2} - \frac{\pi}{4} = -\frac{15\pi}{4}$

3-130.

a. The equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$. Here, the center is $(0,0)$ and the radius is $r = 1$ so the equation of the circle is: $x^2 + y^2 = 1$.

b. Let $x = \frac{1}{2}$ then $\left(\frac{1}{2}\right)^2 + y^2 = 1$

$$y^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$y = \pm\sqrt{3/4} = \pm\frac{\sqrt{3}}{2}$$

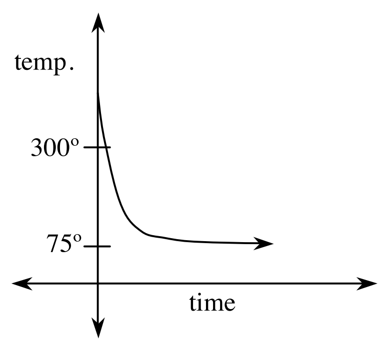
3-131.

$f(x) = |x-3| + 4$ can be approximated with:

$$f(x) = \begin{cases} (x-3)+4 & \text{for } x \geq 3 \\ -(x-3)+4 & \text{for } x < 3 \end{cases} \text{ or } f(x) = \begin{cases} x+1 & \text{for } x \geq 3 \\ -x+7 & \text{for } x < 3 \end{cases}$$

Closure

Merge Problem



3-132.

a. See graph at right.

b. $f(t) = am^t + k$ where k is where equilibrium value of the pie, i.e. the room temperature. $k = 75°\text{F}$

c. Using the form $f(t) = y = am^t + 75$ and the points $(t,y) = (2,323)$ and $(t,y) = (5,288)$ we can find values for a and m.

$323 = am^2 + 75$
$\Rightarrow 323 - 75 = am^2$
$248 = am^2$
$248m^{-2} = a$

$288 = am^5 + 75$
$\Rightarrow 288 = 248m^{-2}m^5 + 75$
$288 - 75 = 248m^{5-2}$
$213 = 248m^3$
$\frac{213}{248} = m^3$
$m = \left(\frac{213}{248}\right)^{1/3}$

$248m^{-2} = a$
$248\left(\left(\frac{213}{248}\right)^{1/3}\right)^{-2} = a$
$248\left(\frac{213}{248}\right)^{-2/3} = a$

$m = 0.95055$ and $a = 274.5$, thus: $f(t) = 274.5(0.95055)^t + 75$

d. When $t = 0$, $f(0) = 274.5(0.95055)^0 + 75 = 274.5 + 75 = 349.5$

e. When the internal temperature of the pie reaches $120°$.

$120 = 274.5(0.95055)^t + 75$

$120 + 75 = 274.5(0.95055)^t$

$\frac{45}{274.5} = (0.95055)^t$

$0.164 = (0.95055)^t$

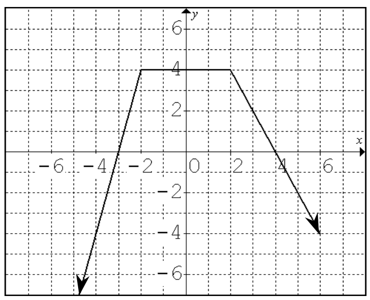
$\log 0.164 = t \log(0.95055)$

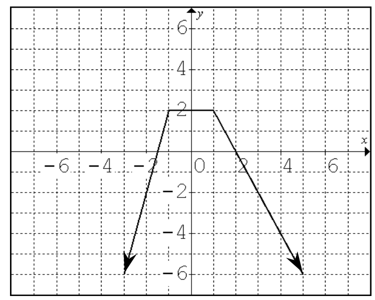
$\frac{\log 0.164}{\log(0.95055)} = t$

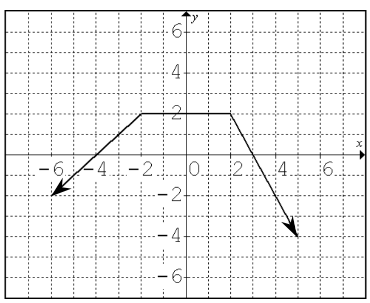
$t = 35.7$ minutes

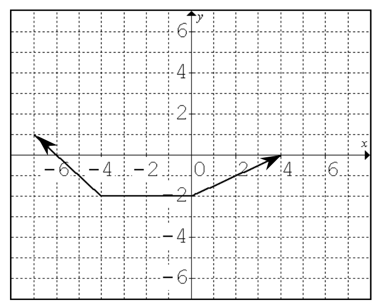
Closure Problems

CL 3-133.

a. 

b. 

c. 

d. 

CL 3-134.

a. As x increases from 4 to 6, y decreases from 80 to 20, thus the function is decreasing.

b. The function is quartered as x increases 2 units, thus it is halved as x increases by 1 unit. The base or multiplier for the exponential function is $\frac{1}{2}$.

c. Using the form of an exponential function $y = m\left(\frac{1}{2}\right)^x$ and either point $(x, y) = (4, 80)$ or $(x, y) = (6, 20)$ we can find m.

$80 = m\left(\frac{1}{2}\right)^4$

$80 = \frac{1}{16} \cdot m$

$\Rightarrow 80 \cdot 16 = m$

$1280 = m$

$y = 1280\left(\frac{1}{2}\right)^x$

CL 3-135.

a. $f(x) = 3(2)^{x+2}$

$= 3 \cdot 2^x \cdot 2^2$

$= 3 \cdot 2^2 \cdot 2^x = 12 \cdot 2^x$

b. $f(x) = 90(1.5)^{x-2}$

$= 90 \cdot 1.5^x \cdot 1.5^{-2}$

$= 90 \cdot 1.5^{-2} \cdot 1.5^x = 40 \cdot 1.5^x$

c. $f(x)=\frac{4}{3}(3)^{2x+1}$

$=\frac{4}{3}\cdot 3^{2x}\cdot 3=\frac{4}{3}\cdot(3^2)^x\cdot 3$

$=4\cdot 9^x$

d. $f(x)=64(4)^{1/2x-2}$

$=64\cdot 4^{1/2x}\cdot 4^{-2}$

$=64\cdot(4^{1/2})^x\cdot\frac{1}{16}=4\cdot 2^x$

CL 3-136.

a. $f(x)=(x-3)^3+2$. Rewrite as $y=(x-3)^3+2$. Switch x and y and solve for y:

$$x=(y-3)^3+2$$
$$x-2=(y-3)^3$$
$$\sqrt[3]{x-2}=y-3$$
$$\sqrt[3]{x-2}+3=y=f^{-1}(x)$$

b. $g(x)=\frac{2x-1}{x+1}$ can be rewritten as $y=\frac{2x-1}{x+1}$. Switch x and y and solve for y:

$$x=\frac{2y-1}{y+1}$$
$$x(y+1)=2y-1$$
$$xy+x=2y-1$$
$$x+1=2y-xy$$
$$x+1=y(2-x)$$
$$\frac{x+1}{2-x}=y=g^{-1}(x)$$

c. $h(x)=5\cdot 2^x$ can be rewritten as $y=5\cdot 2^x$. Switch x and y and solve for y:

$$x=5\cdot 2^y$$
$$\frac{x}{5}=2^y$$
$$\log\frac{x}{5}=\log 2^y$$
$$\log\frac{x}{5}=y\log 2$$
$$\frac{\log x/5}{\log 2}=y=h^{-1}(x)$$

d. $k(x)=\log_3(x-5)$ can be rewritten as $y=\log_3(x-5)$. Switch x and y and solve for y:

$$x=\log_3(y-5)$$
$$3^x=y-5$$
$$3^x+5=y=k^{-1}(x)$$

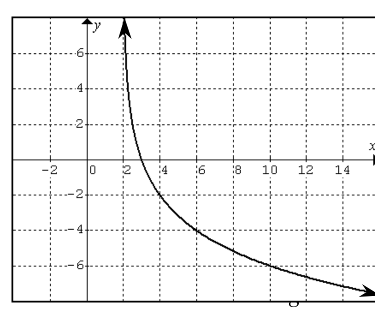
CL 3-137.

a. $\log_2 16=4$ because $2^4=16$.

b. $\log_3\left(\frac{1}{9}\right)=-2$ because $3^{-2}=\frac{1}{9}$

c. $\log_5\sqrt{5}=\frac{1}{2}$ because $5^{1/2}=\sqrt{5}$

CL 3-138.



a. Horizontal shift of 2 units to the right, vertical stretch by a factor of 2, reflection over x-axis.

b. Domain: $(2,\infty)$

c. See graph at right.

CL 3-139.

a. $\log_3 x - \log_3(x-2) = 2$

$$\log_3 \tfrac{x}{x-2} = 2$$
$$3^2 = \tfrac{x}{x-2}$$
$$9(x-2) = x$$
$$9x - 18 = x$$
$$8x = 18$$
$$x = \tfrac{18}{8} = \tfrac{9}{4}$$

b. $3\log_2(x) + \log_2(27) = \log_5(125)$

First notice that $\log_5 125 = 3$.

$$3\log_2 x + \log_2 27 = 3$$
$$\log_2 x^3 + \log_2 27 = 3$$
$$\log_2 27x^3 = 3$$
$$2^3 = 27x^3$$
$$\tfrac{8}{27} = x^3$$
$$\left(\tfrac{8}{27}\right)^{1/3} = (x^3)^{1/3} = x$$
$$x = \tfrac{2}{3}$$

CL 3-140.

a. $20(1.05)^x - 50 = 250$

$$20(1.05)^x = 300$$
$$(1.05)^x = 15$$
$$x = \log_{1.05} 15 = \frac{\log 15}{\log 1.05} = 55.506$$

b. $15(x-2)^{3.7} = 50$

$$(x-2)^{3.7} = \tfrac{50}{15} = \tfrac{10}{3}$$
$$\left((x-2)^{3.7}\right)^{1/3.7} = \left(\tfrac{10}{3}\right)^{1/3.7}$$
$$x - 2 = 1.385$$
$$x = 3.385$$

c. $3\log_4(x-2) = 6$

$$\log_4(x-2) = 2$$
$$4^2 = x - 2$$
$$16 + 2 = x$$
$$x = 18$$

CL 3-141.

a. Using the Law of Sines, $\frac{1}{\sin 90} = \frac{\text{base}}{\sin 45}$

$$\text{base} = \frac{\sin 45}{\sin 90} = \frac{\sqrt{2}}{2}$$

The process repeats for the height of the triangle.

b. $\frac{1}{\sin 90} = \frac{\text{base}}{\sin 60}$

$$\text{base} = \frac{\sin 60}{\sin 90} = \frac{\sqrt{3}}{2}$$

$\frac{1}{\sin 90} = \frac{\text{height}}{\sin 30}$

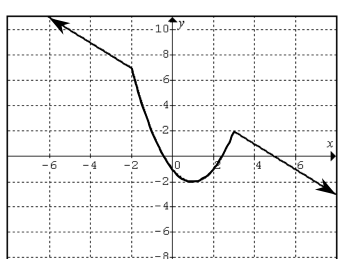
$$\text{height} = \frac{\sin 30}{\sin 90} = \frac{1}{2}$$

CL 3-142.

a. $\frac{\pi}{6}$

b. $\frac{\pi}{4}$

c. $\frac{\pi}{3}$



CL 3-143.

See graph at right.